Numeric Response Questions

Area Under the Curve

Q.1 Find the area between the curve $y = 1 - |x|$ and x-axis.

Q.2 Find the area of the region enclosed between the curve $x^2 = 2y$ and the straight line $y = 2$.

Q.3 Find the common area of the curves $y = \sqrt{x}$ and $x = \sqrt{y}$,

Q.4 Find the area bounded by the curve $y = \sin\ x$, x-axis and the lines $x = 0$ and $x = \pi$.

Q.5 Find the area between the parabola $x^2 = 4y$ and line $x = 4y - 2$.

Q.6 Find the area of the region bounded by curves $y = |x - 1|$ and $y = 3 - |x|$.

Q.7 Find the area bounded by the curve $y = x^3$, x-axis and ordinates $x = -2$ and $x = 1$.

Q.8 Find the area of the figure bounded by $y^2 = 9x$ and $y = 3x$.

Q.9 If the areu of the region $\{(x, y): x^2 + y^2 \leq 1 \leq x + y\}$ is $\frac{\pi}{4} - k$ then find k.

Q.10 If area bounded by the curve $xy^2 = a^2(a - x)$ and the $y \cdot axis\ is\ ka^2$ then find k.

Q.11 If the area bounded by the curves $y = e^x, y = e^{-x}$ and $y = 2$, is $2\log\left(\frac{k}{e}\right)$ then find k.

Q.12 Find the area bounded by the eurves $y = |x - 2|, x = 1, x = 3$ and x-axis,

Q.13 If the area bounded by the curves $y = \sin\ x, y = \cos\ x$ and y-axis in first quadrant is $(\sqrt{k} - 1)$ then find k.

Q.14 Find the area bounded by region $\{(x, y): |x| \geq y \geq x^2\xi$.

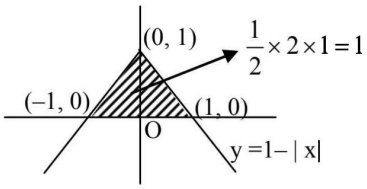
Q.15 Find the area bounded by loop of $|y| = \sin\ x$ for $0 \leq x \leq \pi$.

ANSWER KEY

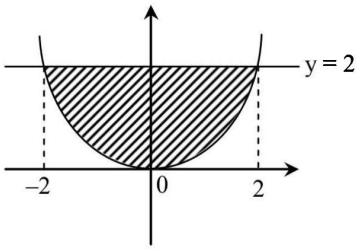
1. 1.00 **2.** 5.33 **3.** 0.33 **4.** 2.00 **5.** 1.12 **6.** 4.00 **7.** 4.25
8. 0.5 **9.** 0.5 **10.** 3.14 **11.** 4.00 **12.** 1.00 **13.** 2.00 **14.** 0.33
15. 4.00

Hints & Solutions

1.

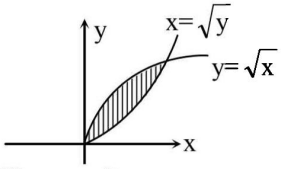


2.



$$\text{Area} = 8 - \int_{-2}^{2}\left(\frac{x^2}{2}\right)dx$$

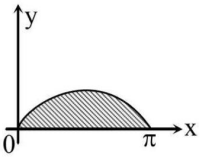
3.



Use result,

$$\text{Area} = \frac{16}{3}ab = \frac{16}{3}\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) = \frac{1}{3}$$

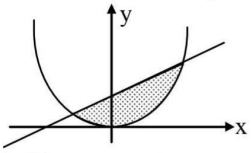
4.



$$\text{Area} = \int_0^{\pi}\sin x\,dx = (-\cos x)_0^{\pi}$$

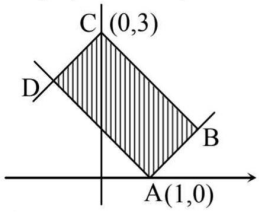
$$= -[\cos\pi - \cos 0] = 2$$

5. $x^2 = x + 2 \quad \Rightarrow x = -1, 2$



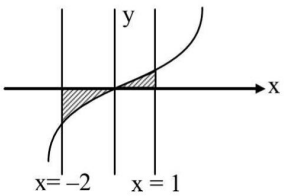
$$\text{Area} = \int_{-1}^{2}\left[\frac{x+2}{4} - \frac{x^2}{4}\right]dx = \frac{9}{8}$$

6. $AB = 2/\sqrt{2}$, $BC = 4/\sqrt{2}$.



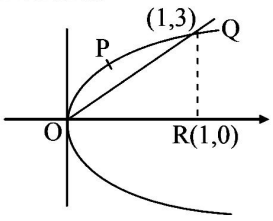
$\therefore$ Area = 4

7.



$$\text{Area} = \left|\int_{-2}^{0} x^3\,dx\right| + \int_0^1 x^3\,dx = \frac{17}{4}$$

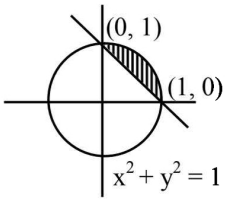
8. Required area



= area OPQRO – area ΔOQR

$= \int_0^1 \sqrt{9x}\, dx - \frac{1}{2}\times 1\times 3 = 3\,\frac{2}{3}x^{3/2}\Big|_0^1 - \frac{3}{2} = \frac{1}{2}$

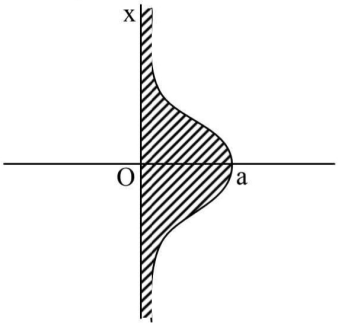
9.



$= \frac{\pi\times 1^2}{4} - \frac{1}{2}\times 1\times 1$

10. $y^2 = a^2\frac{(a-x)}{x}$

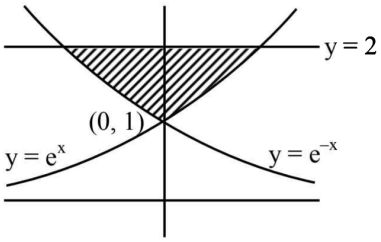
$y = \pm\, a\sqrt{\frac{a-x}{x}}$



$= 2\int_0^a a\sqrt{\frac{a-x}{x}}\, dx$

Put $x = a\sin^2\theta$

11.



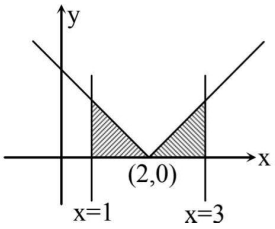
$2\int_1^2 \log y\, dy = 2(y\log y - y)_1^2$

$= 2\,((2\log 2 - 2) - (0 - 1))$

$= 2\,(2\log 2 - 1)$

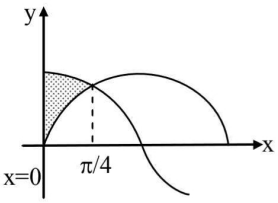
$= 2\left(\log\frac{4}{e}\right)$

12.



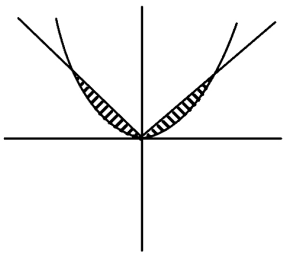
Area $= 2\left\{\frac{1}{2}(1)(1)\right\} = 1$

13.



Area $= \int_0^{\pi/4}(\cos x - \sin x)dx = = \sqrt{2} - 1$

14.



Required area

$= 2\int_0^1 (x - x^2)\, dx$

$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$

$= 2\left[\frac{1}{2} - \frac{1}{3}\right] = 2\times\frac{1}{6} = \frac{1}{3}$

15. $A = 2\int_0^{\pi}\sin x\, dx = 4$